Calculating a Calibration Factor

When using an RF Power Meter with an RF Power Sensor to make an RF Power Measurement the user must know that the measurement is accurate and there is traceability to a known standard. All diode, thermo-electric, and thermistor power sensors have calibration factors associated to particular frequencies that are used to insure an accurate power measurement. Technicians and engineers use these calibration factors when making measurements; but where do these calibration factors really come from?

Calibration factor of a terminating power sensor, if it is a DC-substitution sensor, relates the change in DC substituted power to the total RF power *incident* on the sensor. For this purpose, incident means all of the RF propagating toward the sensor reference plane, including power that is subsequently reflected. On the signal flow diagram, the incident power is $P_i = |a_l|^2$.

Calibration factor of a feed-through power calibration setup, again if the monitor is a DC-substitution sensor, relates the change in DC substituted power in the monitor to the power delivered out of the DUT port into a load of exactly the nominal characteristic impedance of the system, or Z_0. If we think of the feed-through DUT port as the output of a leveled generator, then the output into a perfect load is $P_{g_{z0}}$.

So for a terminating sensor (using the "M" subscript following Weinschel part numbering):

$$k_M = \frac{P_{SubM}}{P_i}$$

And for a feed-through stand, (using "F" to indicate "feedthrough"):

$$k_F = \frac{P_{SubF}}{P_{g_{Z0}}}$$

Where:
k_M = Calibration factor of the Terminating Mount
k_F = Calibration factor of the Feed-through Mount
P_{SubM} = Power measured terminating mount
P_{SubF} = Power measured Feed-through mount

In all calibrations, calibration factors are transferred from a terminating reference, to a feed-through stand, and then into the DUT. In some procedures, this transfer occurs all in one session, and in others time is saved by transferring into the feed-through once and then using that feed-through calibration factor to calibrate DUT for some time.

In a perfect world, the terminating sensor would present a perfect load, and then by reorganizing the above two definitions with incident and output power equal, we would have,

$$k_M = k_F \frac{P_{SubM}}{P_{SubF}}$$

Because power sensors are always imperfect loads, additional analysis and sometimes correction is required.

Figure 1 shows a signal flow diagram of two one-port devices connected together. The left-hand port is the "output", or "generator" port and is designated on the diagram using the subscript, "g". The right-hand port is the "input", or "load" port and is designated using the subscript "l". It's clear from the diagram that port reflections denominated by the Gamma vectors generally cause the power that the generator port would output into an ideal load, $P_{g_{z0}}$, to not be equal to the power P_i, incident on the load. We really have to take into account the combination of reflections, or "port match" to come up with a general understanding of calibration transfer.

Start with the signal flow diagram. In this case, a Generator "g" is represented by an ideal source "s", and its port reflection, and the sensor is termed the load, "l".

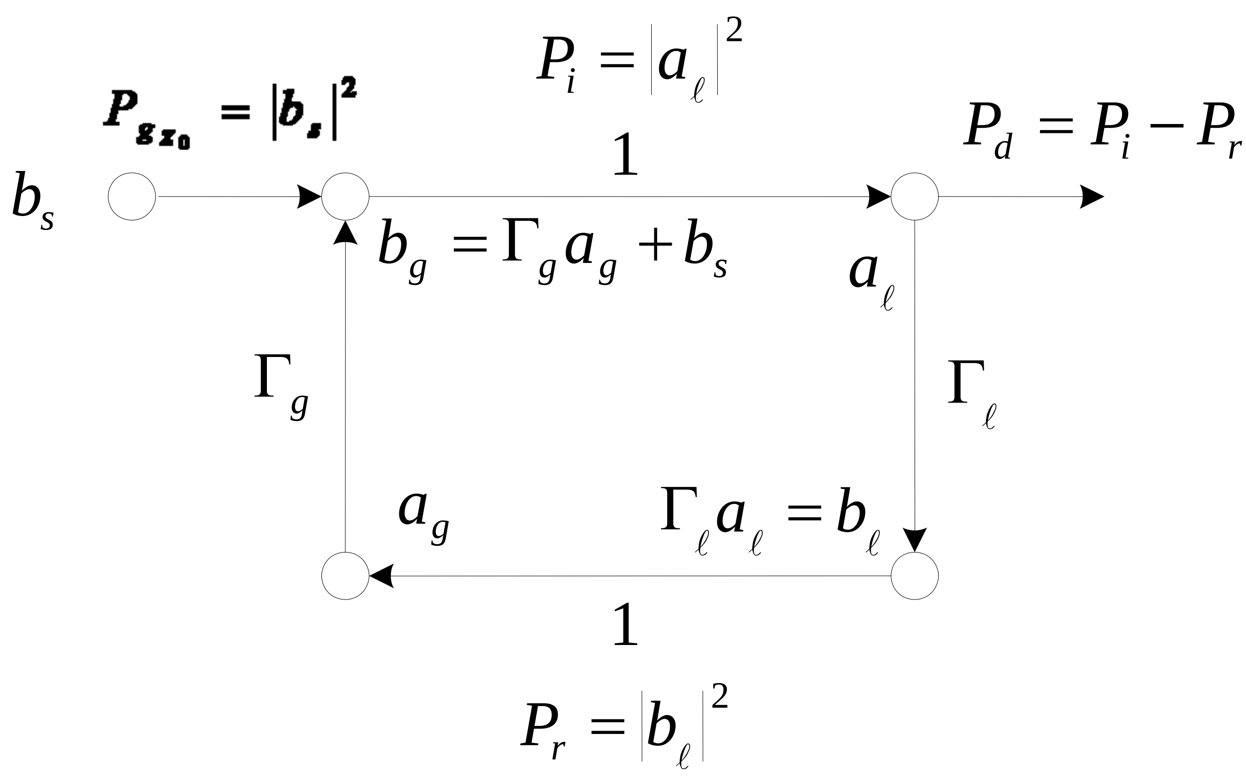

Figure 1 - Signal Flow Diagram

From the diagram,

$$b_g = b_s + \Gamma_g a_g$$

But also,

$$a_g = b_l = \Gamma_l a_l = \Gamma_l b_g$$

So substituting the second equation into the first:

$$b_g = b_s + \Gamma_l \Gamma_g b_g$$

And then re-arrange to collect b_g:

$$b_g = \frac{b_s}{1 - \Gamma_l \Gamma_g}$$

Now rewrite as power, substituting in $P_{g_{z0}} = |b_s|^2$, and $P_i = |b_g|^2$:

$$P_i = \frac{P_{g_{z0}}}{|1 - \Gamma_l \Gamma_g|^2}$$

Armed with this last result, and calling the Feed-through stand (F subscript) the "generator" (g subscript) and terminating sensor (M subscript) the "load" (l subscript), and substituting in the definitions for cal factor from earlier, we get the more general equation for transferring between a feed-through and a terminating sensor:

$$k_M = k_F \frac{P_{SubM}}{P_{SubF}} |1 - \Gamma_M \Gamma_F|^2$$

Where:
k_M = Calibration factor of the Terminating Mount
k_F = Calibration factor of the Feed-through Mount
P_{SubM} = Power measured terminating mount
P_{SubF} = Power measured Feed-through mount
Γ_M = Gamma Correction full vector data Terminating Mount
Γ_F = Gamma Correction full vector data Feed-through Mount

Now in this general equation, the Gamma terms are the reflection scattering parameter of the respective port noted in the subscript. Gamma is a complex vector with scalar values denoting the real and imaginary magnitudes:

$$\Gamma \equiv \rho\angle\phi = \rho\cos\phi + \mathrm{i}\rho\sin\phi$$

In the general transfer equation, the term, $|1-\Gamma_M\Gamma_F|^2$ is the scalar "gamma correction" or "port match" term. Inside the absolute value brackets, however, is a *vector* subtraction. Expanding out to make the angles explicit, this becomes:

$$|1-\rho_M\rho_F\cos(\phi_M+\phi_F) - i\,\rho_M\rho_F\sin(\phi_M+\phi_F)|^2$$

Where the *i* represents $\sqrt[2]{-1}$, or the "imaginary" component.

The absolute value, or length of a vector, is given by the Pythagorean formula, which is the square root of the square of the magnitudes of the real and imaginary components. It's convenient that we are looking for the square of the magnitude, so we don't have to worry about the square root part. Our correction term becomes the scalar,

$$(1-\rho_M\rho_M\cos(\phi_M+\phi_F))^2 + (\rho_M\rho_F\sin(\phi_M+\phi_F))^2$$

When the squares are evaluated, this expands to:

$$1-2\rho_M\rho_F\cos(\phi_M+\phi_F) + {\rho_M}^2{\rho_F}^2\cos^2(\phi_M+\phi_F) + {\rho_M}^2{\rho_F}^2\sin^2(\phi_M+\phi_F)$$

Noting that if we collect the two terms beginning with ${\rho_M}^2{\rho_F}^2$, we get,

$$1-2\rho_M\rho_F\cos(\phi_M+\phi_F) + {\rho_M}^2{\rho_F}^2(\cos^2(\phi_M+\phi_F)+\sin^2(\phi_M+\phi_F))$$

The term, $\cos^2(\phi_M+\phi_F)+\sin^2(\phi_M+\phi_F)$ is always identically equal to 1, so the final simplified equation becomes,

$$1-2\rho_M\rho_F\cos(\phi_M+\phi_F) + {\rho_M}^2{\rho_F}^2$$

Or combining this result with the general transfer equation,

$$k_M = k_F\frac{P_{SubM}}{P_{SubF}}(1-2\rho_M\rho_F\cos(\phi_M+\phi_F) + {\rho_M}^2{\rho_F}^2)$$

If we look at the scalar result of the mismatch term, the "1" part is what would happen if at least one of the ports was "perfect", or had no reflection. In that case, one of the ρ is zero. The right-most element has magnitude of ρ^4, which is typically so much smaller than the middle term that it can be ignored for most connections.

The middle part, $2\rho_M\rho_F\cos(\phi_M + \phi_F)$, contains the bulk of the impact of port mismatch. Since it is multiplied by k, the sensitivity to this change is equal to k, or about 1. In an *UNCORRECTED* transfer, this part represents the probable error of the transfer. Since we can't know the angles in an uncorrected transfer, we let cos() take its limits of +/- 1, and say that the uncertainty of the uncorrected transfer is $2\rho_M\rho_F$. That is a little pessimistic, because that is worst-case rather than probable, but it's what the industry usually does. Unless the rhos were measured on a scalar analyzer, we have to use manufacturer's worst-case values.

Some common power sensor calibration practices do not always use gamma correction. An argument could probably be made that this was a reasonable practice at lower frequencies. We typically see this at 18 GHz and lower. A value of 0 would be inserted for □□□making that portion of the formula "1".

This would make the formula look like:

$$k_M = k_F \frac{P_{SubM}}{P_{SubF}}$$

This revision of the original formula assumes many things that are not necessarily true. Things that may not necessarily show up at lower frequencies but will certainly show up at higher frequencies where connectors change from the very rugged N-type connector to the more sensitive 3.5 mm and 2.4 mm connectors.

By looking at Figure 2 we can get a practical visualization of the relationship between calibration factor, gamma and effective efficiency.

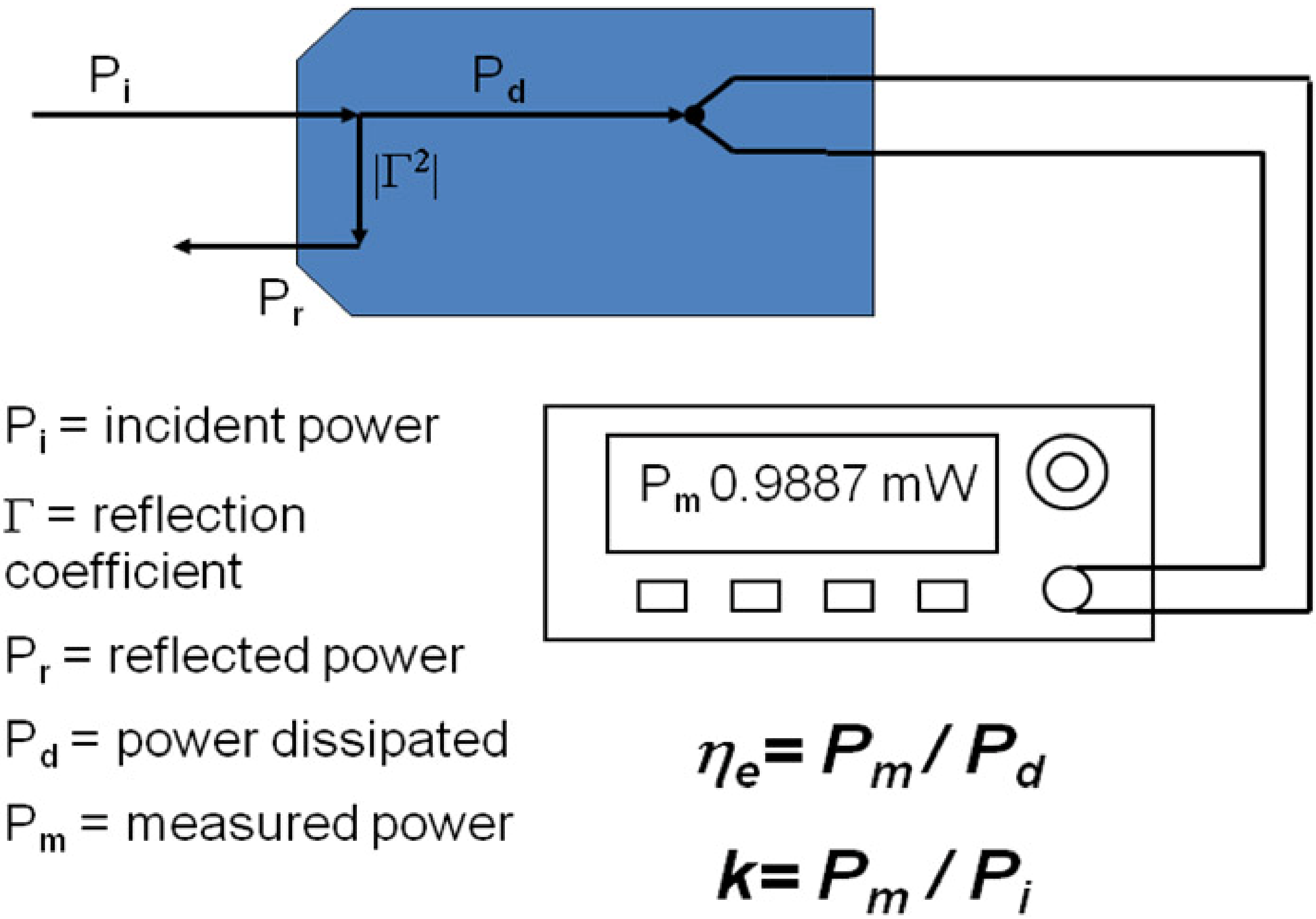

Figure 2 - Relationship between Calibration Factor, Gamma, and Effective Efficiency